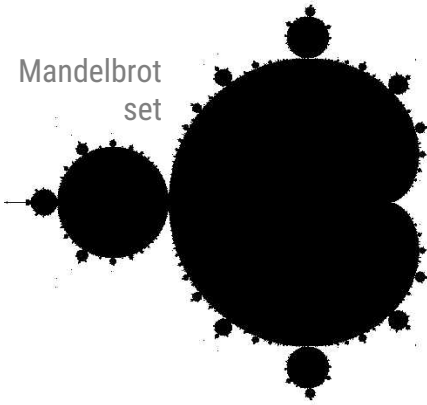


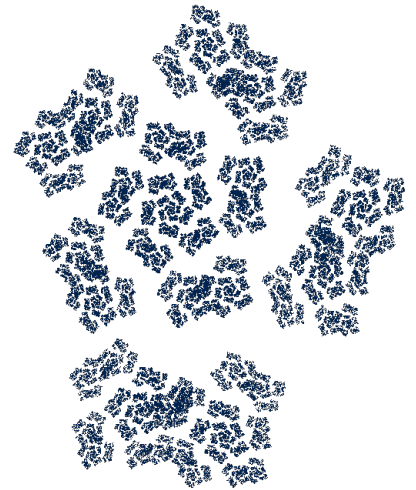
—
Make your own fractal!

Mandelbrot
set



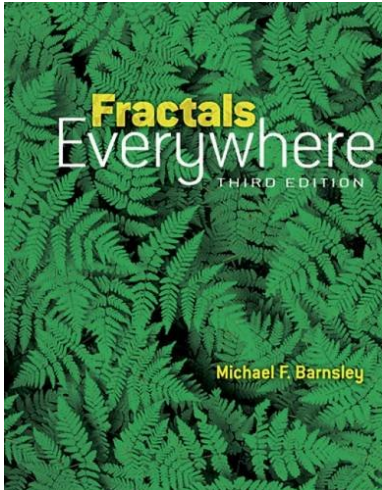
What is a Fractal?

- Self-similar
- Nowhere differentiable
- Non-integer dimension





[nl.wikipedia.org/wiki/Romanesco_\(groente\)](https://nl.wikipedia.org/wiki/Romanesco_(groente))

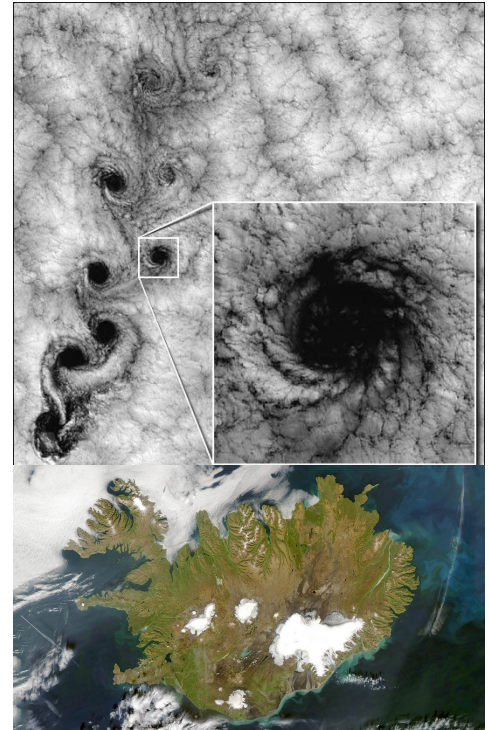


<https://m.media-amazon.com/images/I/61+q6AqRXhL.jpg>



www.pflanzen-deutschland.de/Dryopteris_filix-mas.html

science.nasa.gov/earth/earth-observatory/landsat-7-reveals-large-scale-fractal-motion-of-clouds-625/



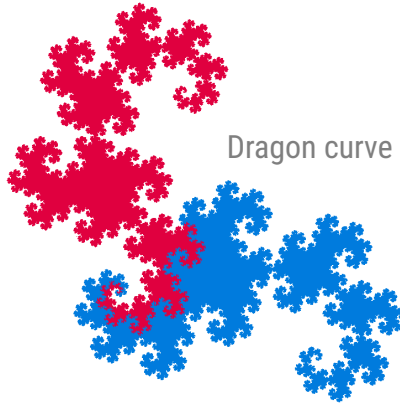
<https://eliasvetter.ch/Island/>

Self-Similar Sets

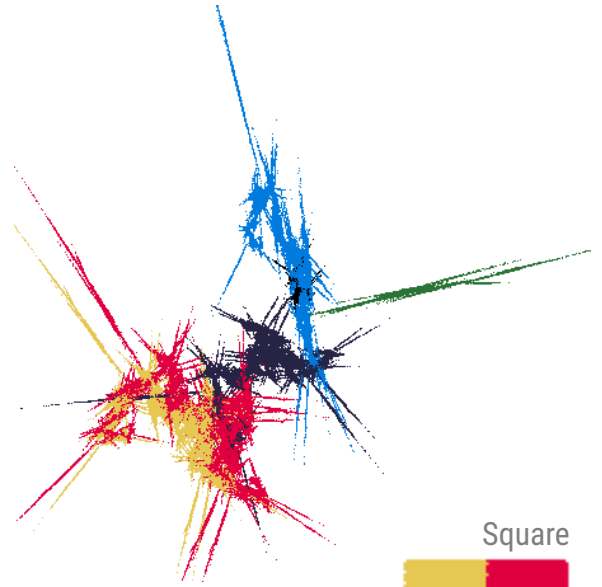
... consist of **finitely many** smaller copies of themselves



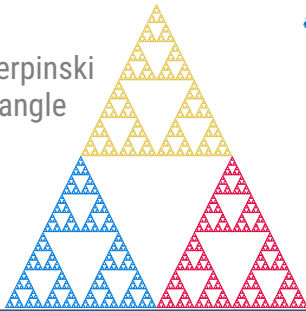
Barnsley fern



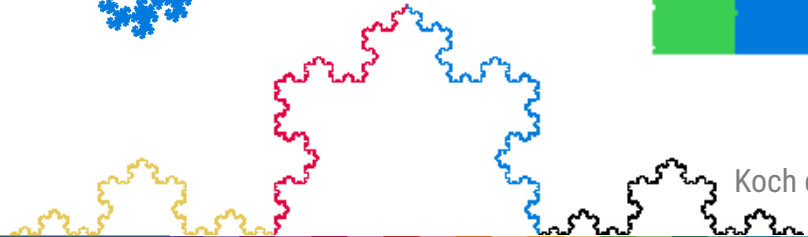
Dragon curve

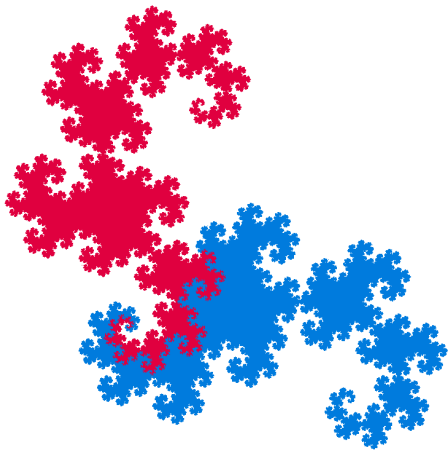


Sierpinski triangle



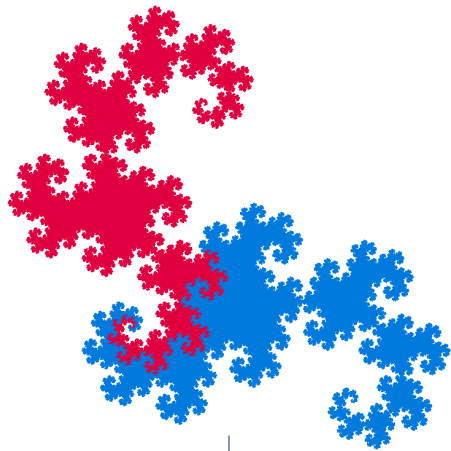
Koch curve



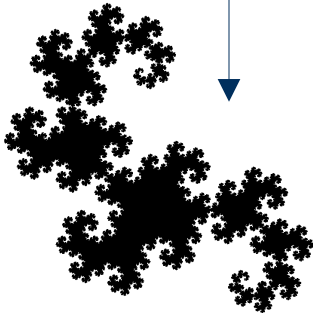


How do we describe a self-similar set?

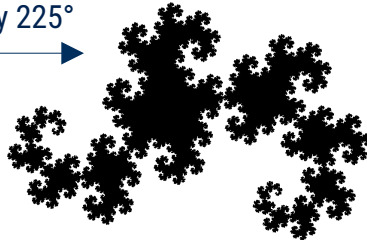
How do we describe a self-similar set?



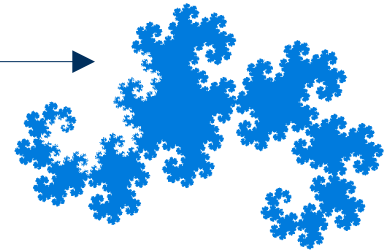
shrink by $1/\sqrt{2}$



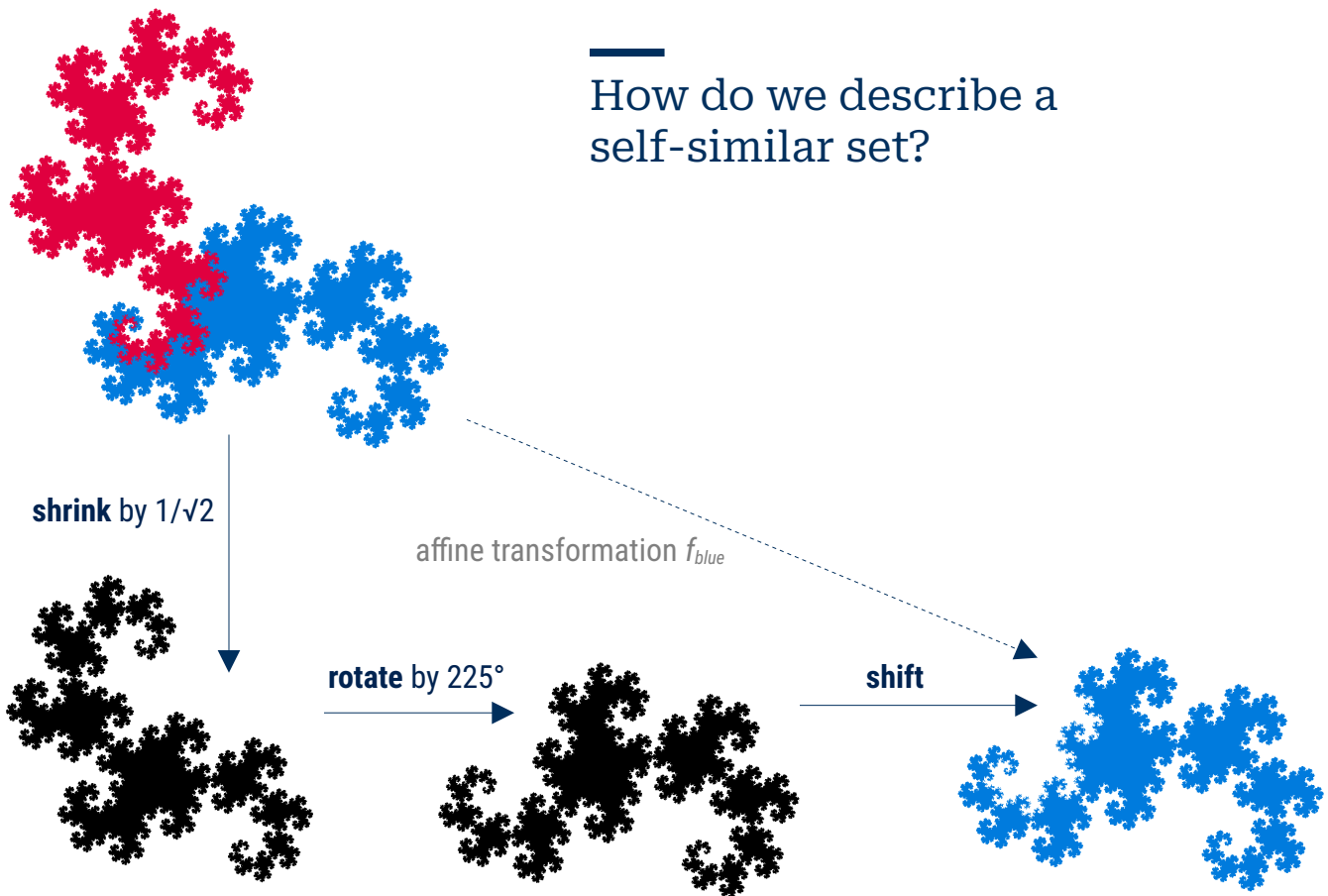
rotate by 225°



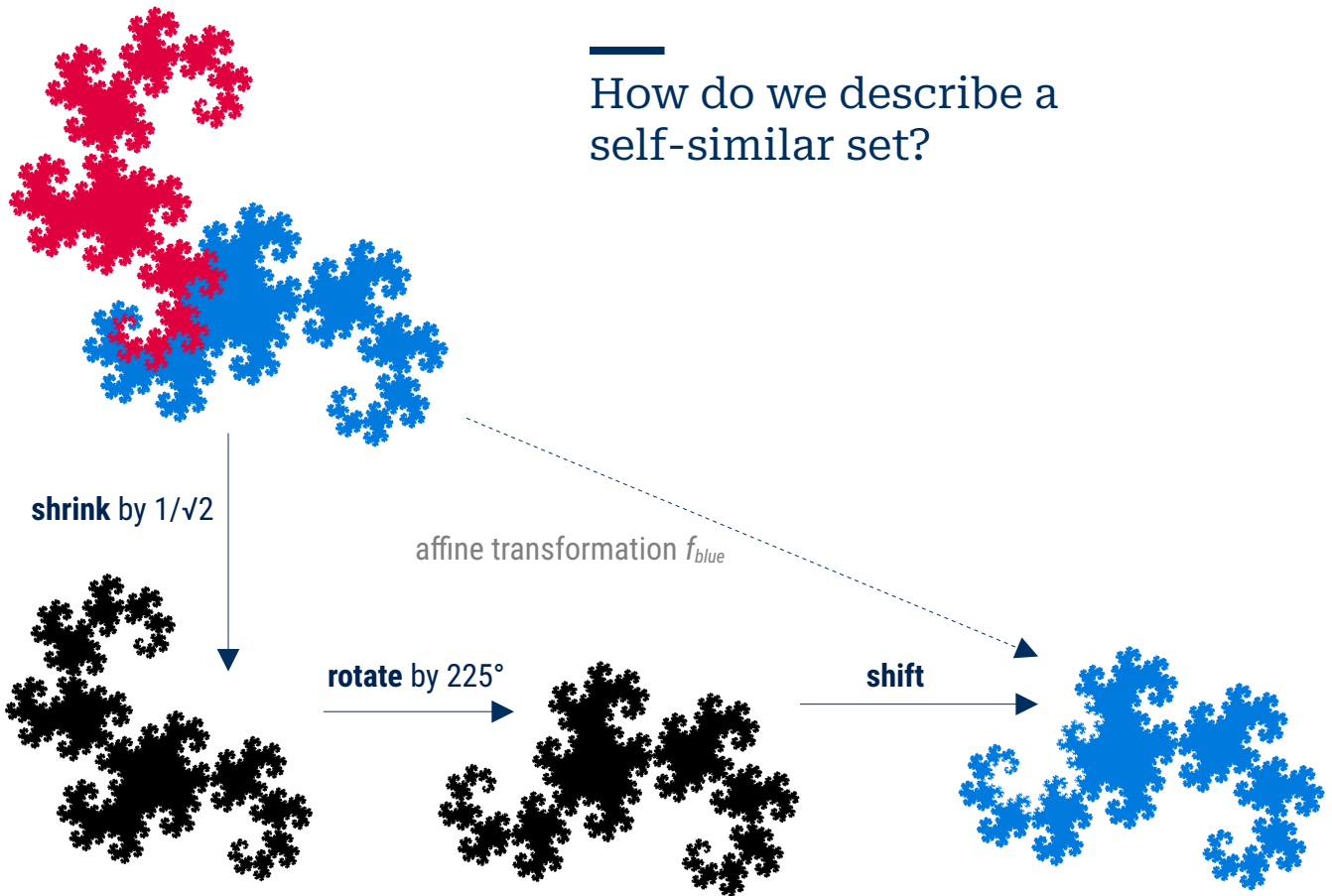
shift



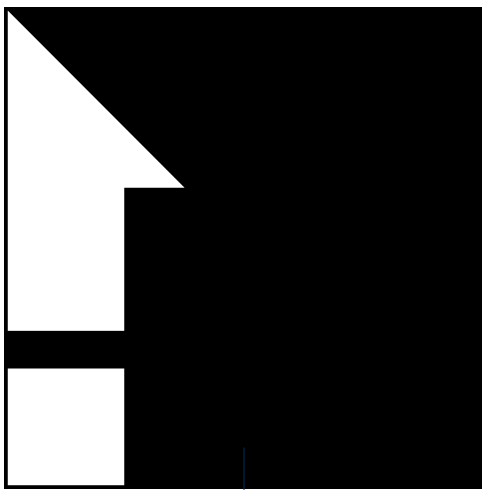
How do we describe a self-similar set?



How do we describe a self-similar set?



How do we describe a self-similar set?



shrink by $1/\sqrt{2}$

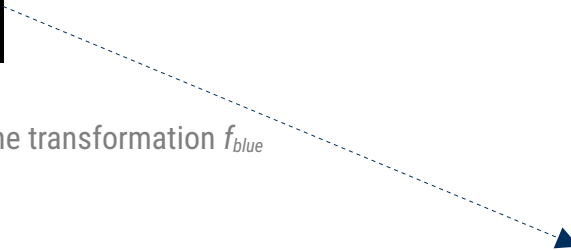


rotate by 225°

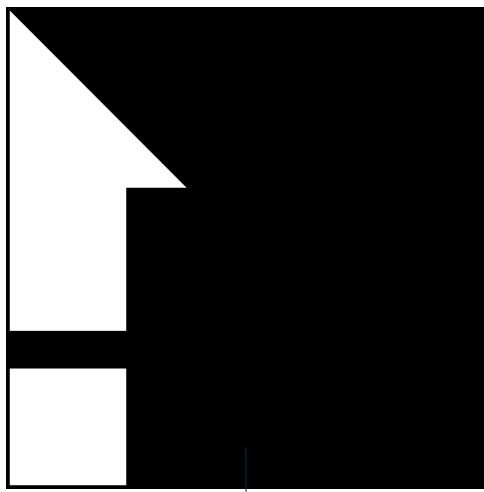


affine transformation f_{blue}

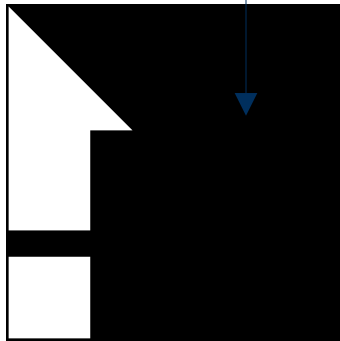
shift



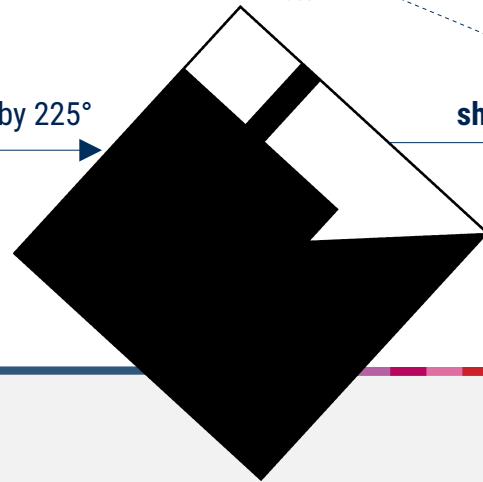
How do we describe a self-similar set?



shrink by $1/\sqrt{2}$

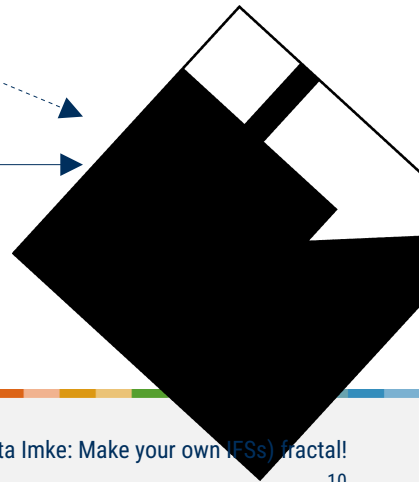


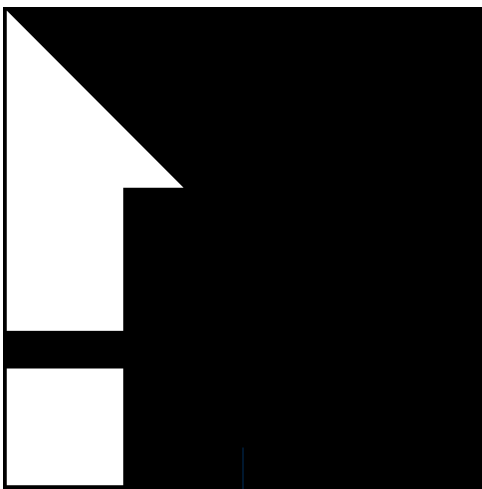
rotate by 225°



affine transformation f_{blue}

shift





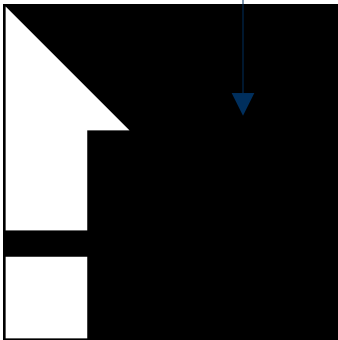
How do we describe a self-similar set?

- For each copy i :
Find an affine transformation f_i with
 $f_i(\text{whole set}) = \text{copy } i$
- All f_i together describe the set

Iterated
Function
System

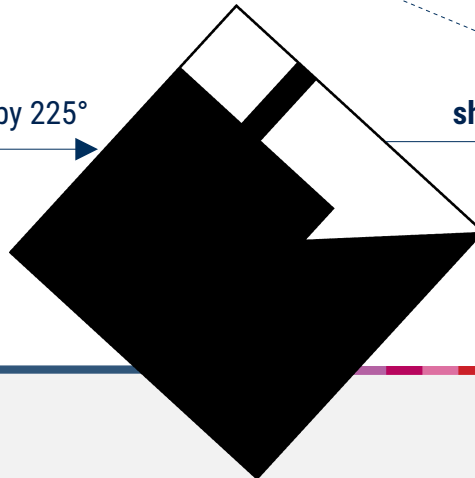
IFS

shrink by $1/\sqrt{2}$

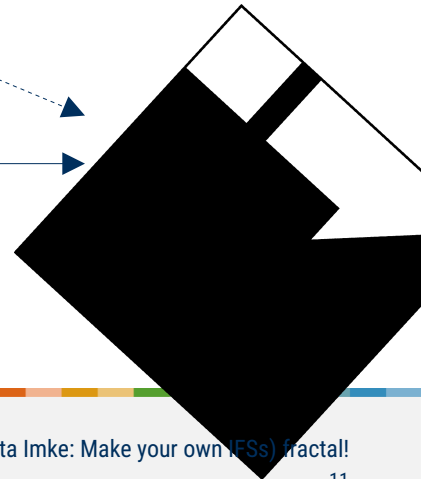


rotate by 225°

affine transformation f_{blue}



shift

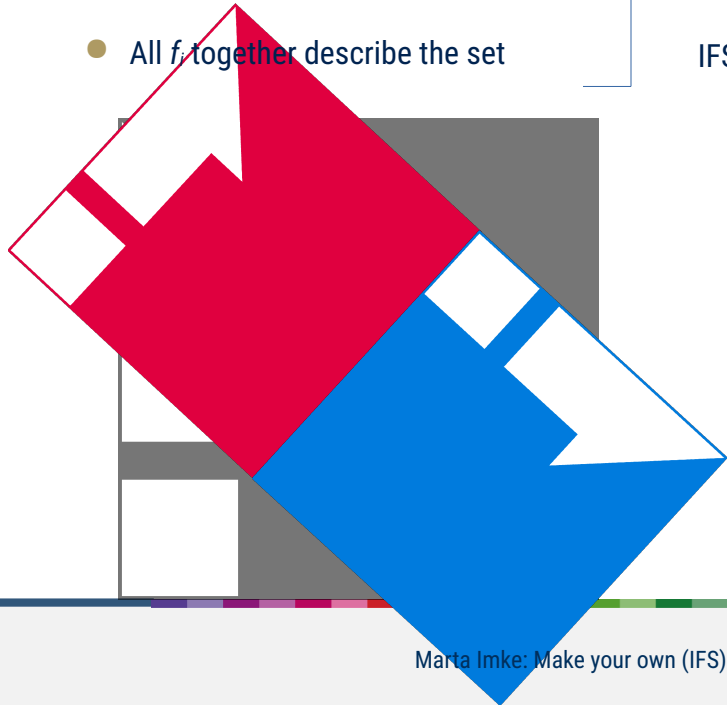
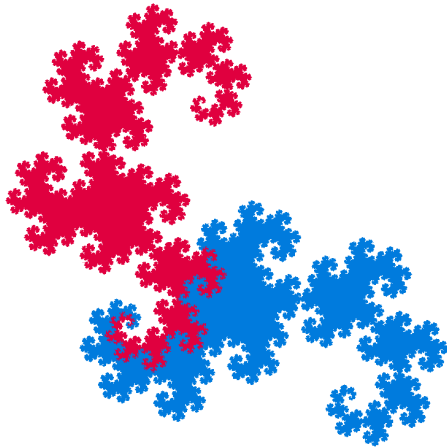


How do we describe a self-similar set?

- For each copy i :
Find an affine transformation f_i with
 $f_i(\text{whole set}) = \text{copy } i$
- All f_i together describe the set

Iterated
Function
System

IFS



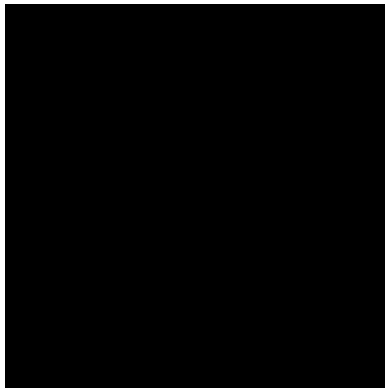
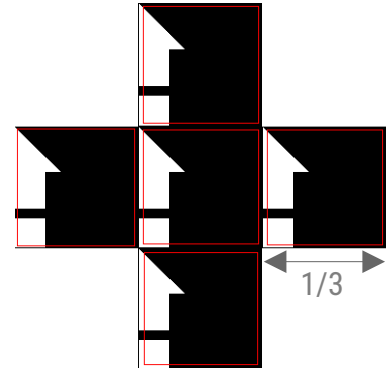
How do I generate a self-similar set?

Given: affine transformations f_i

Copy and paste

- 1) Start with any non-empty image

Chaos Game



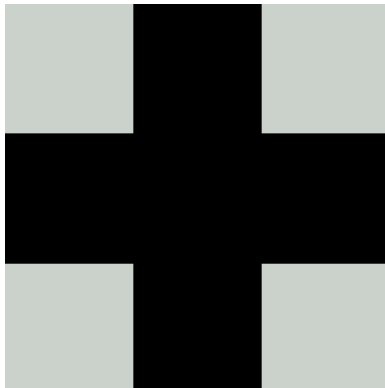
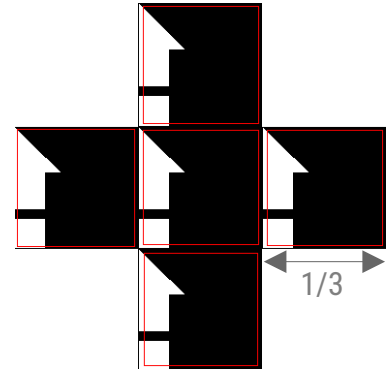
How do I generate a self-similar set?

Given: affine transformations f_i

Copy and paste

- 1) Start with any non-empty image
- 2) For each i :
transform the image

Chaos Game

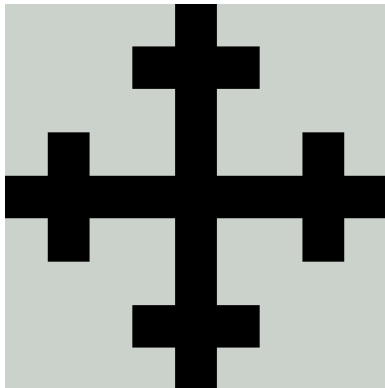


How do I generate a self-similar set?

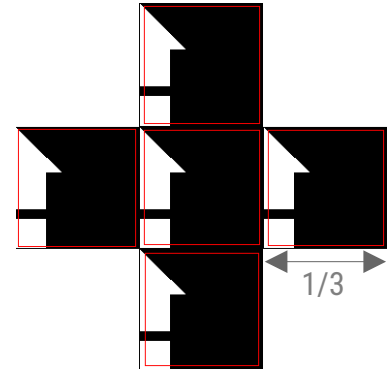
Given: affine transformations f_i

Copy and paste

- 1) Start with any non-empty image
- 2) For each i :
 transform the image
- 3) New image = union of transformed ones
- 4) Repeat



Chaos Game



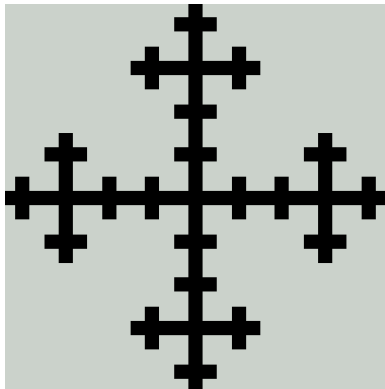
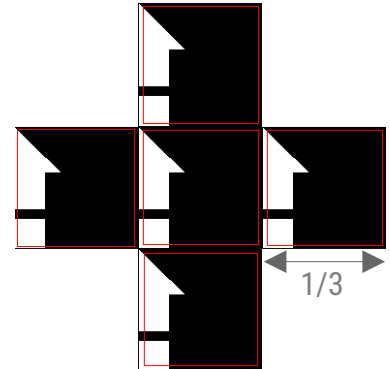
How do I generate a self-similar set?

Given: affine transformations f_i

Copy and paste

- 1) Start with any non-empty image
- 2) For each i :
 transform the image
- 3) New image = union of transformed ones
- 4) Repeat

Chaos Game



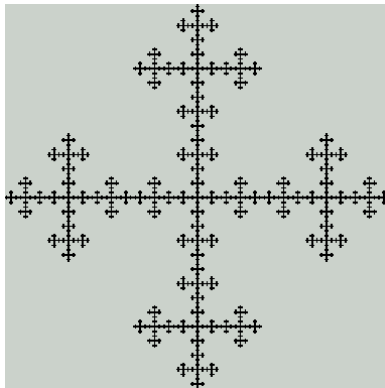
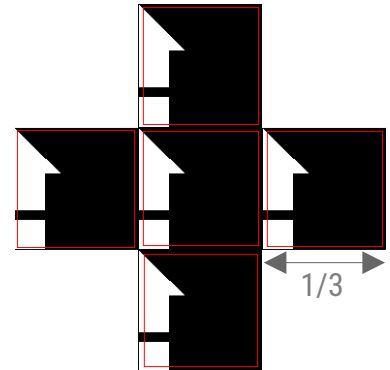
How do I generate a self-similar set?

Given: affine transformations f_i

Copy and paste

Chaos Game

- 1) Start with any non-empty image
- 2) For each i :
 transform the image
- 3) New image = union of transformed ones
- 4) Repeat



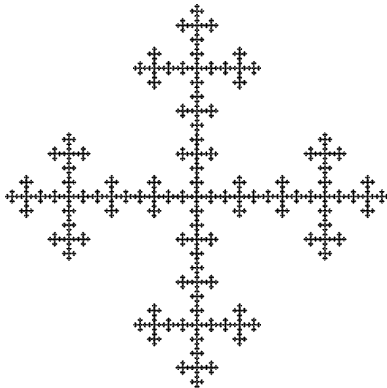
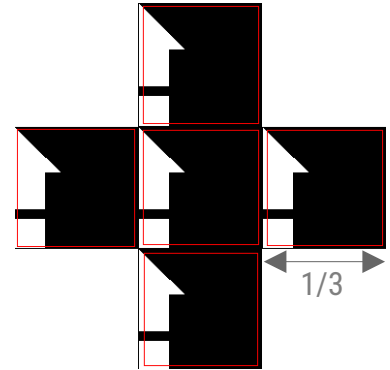
How do I generate a self-similar set?

Given: affine transformations f_i

Copy and paste

Chaos Game

- 1) Start with any non-empty image
- 2) For each i :
 transform the image
- 3) New image = union of transformed ones
- 4) Repeat

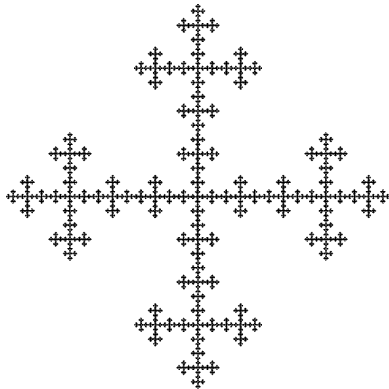


How do I generate a self-similar set?

Given: affine transformations f_i

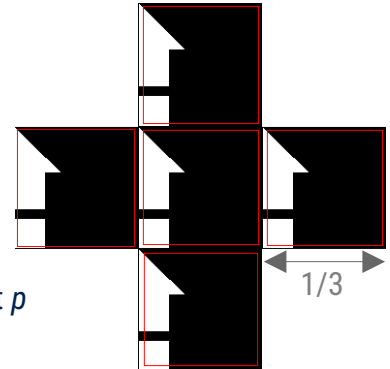
Copy and paste

- 1) Start with any non-empty image
- 2) For each i :
 transform the image
- 3) New image = union of transformed ones
- 4) Repeat



Chaos Game

- 1) Start with any point p
- 2) Draw p

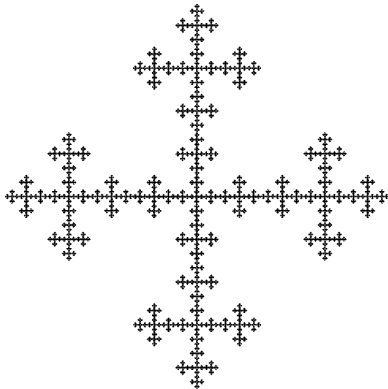


How do I generate a self-similar set?

Given: affine transformations f_i

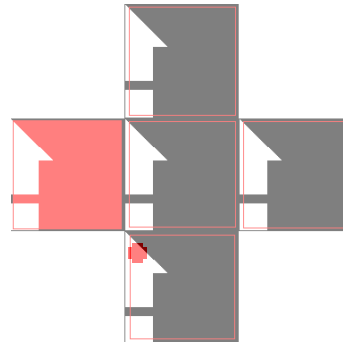
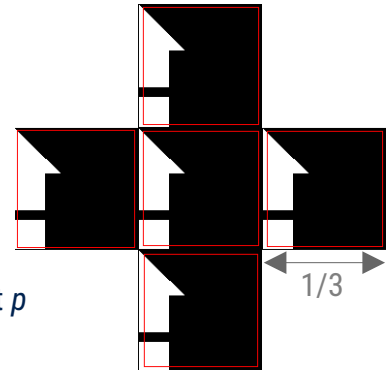
Copy and paste

- 1) Start with any non-empty image
- 2) For each i :
transform the image
- 3) New image = union of transformed ones
- 4) Repeat



Chaos Game

- 1) Start with any point p
- 2) Draw p
- 3) Select a random f_i

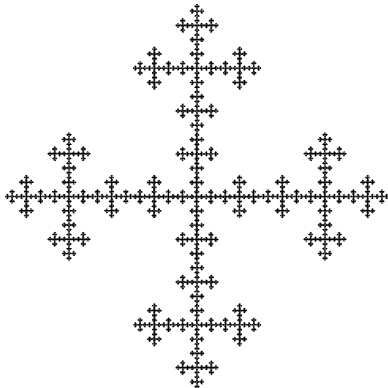


How do I generate a self-similar set?

Given: affine transformations f_i

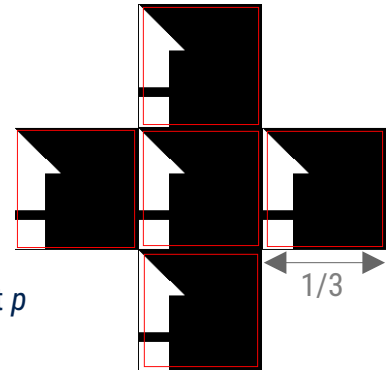
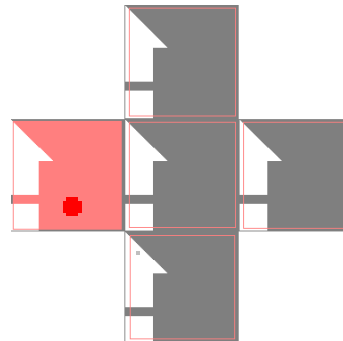
Copy and paste

- 1) Start with any non-empty image
- 2) For each i :
transform the image
- 3) New image = union of transformed ones
- 4) Repeat



Chaos Game

- 1) Start with any point p
- 2) Draw p
- 3) Select a random f_i
- 4) $new_p = f_i(p)$

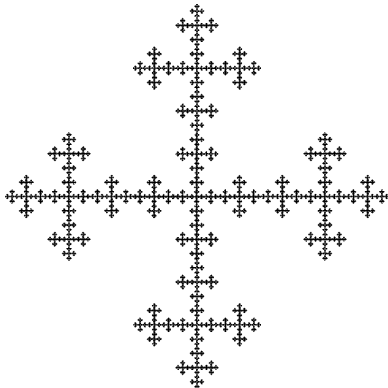


How do I generate a self-similar set?

Given: affine transformations f_i

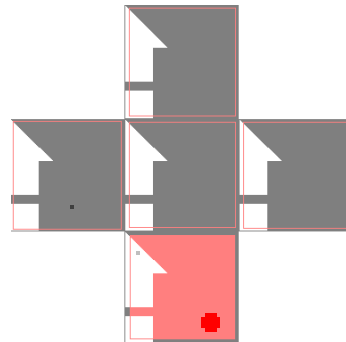
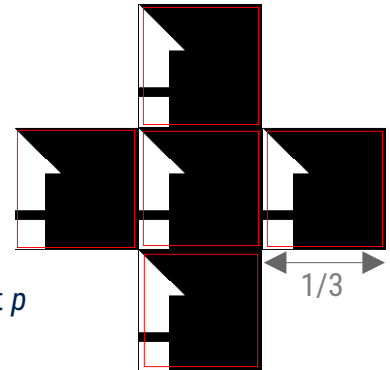
Copy and paste

- 1) Start with any non-empty image
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 transform the image
- 3) New image = union of transformed ones
- 4) Repeat



Chaos Game

- 1) Start with any point p
- 2) Draw p
- 3) Select a random f_i
- 4) $new_p = f_i(p)$
- 5) Repeat



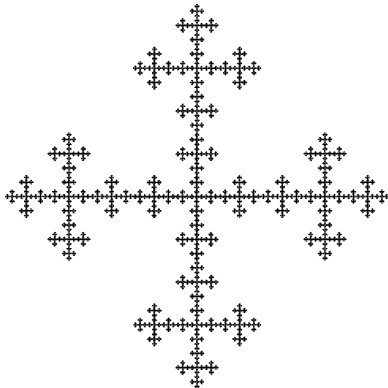
3 points

How do I generate a self-similar set?

Given: affine transformations f_i

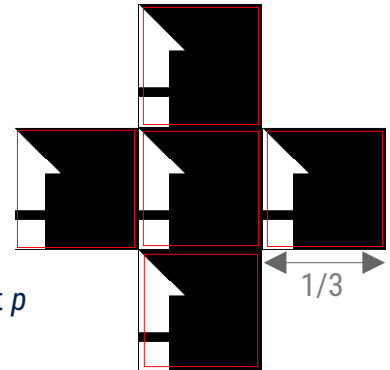
Copy and paste

- 1) Start with any non-empty image
- 2) For each i :
transform the image
- 3) New image = union of transformed ones
- 4) Repeat



Chaos Game

- 1) Start with any point p
- 2) Draw p
- 3) Select a random f_i
- 4) $new_p = f_i(p)$
- 5) Repeat



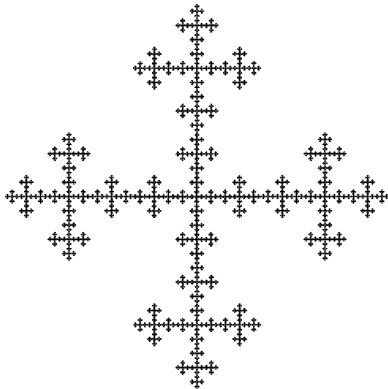
10 points

How do I generate a self-similar set?

Given: affine transformations f_i

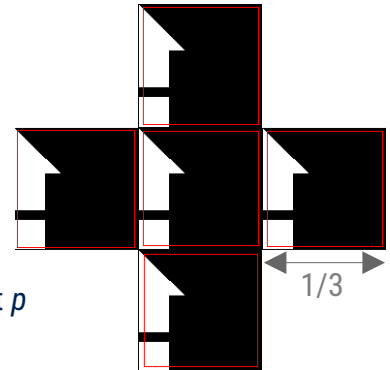
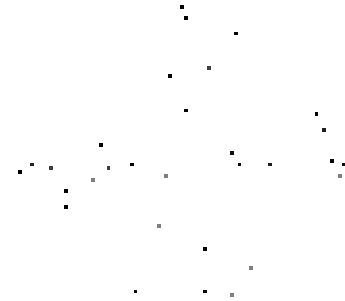
Copy and paste

- 1) Start with any non-empty image
- 2) For each i :
 transform the image
- 3) New image = union of transformed ones
- 4) Repeat



Chaos Game

- 1) Start with any point p
- 2) Draw p
- 3) Select a random f_i
- 4) $new_p = f_i(p)$
- 5) Repeat



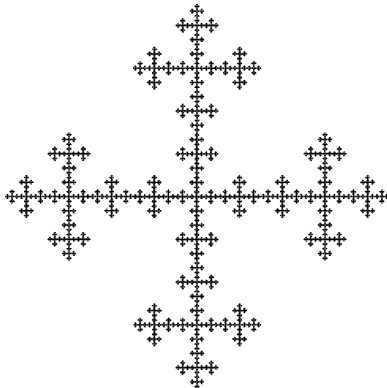
30 points

How do I generate a self-similar set?

Given: affine transformations f_i

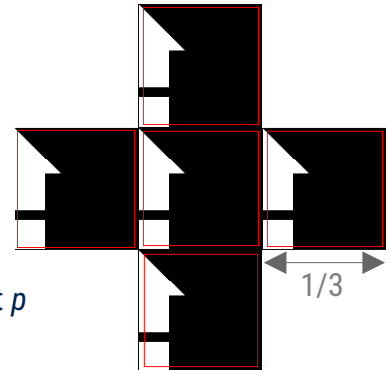
Copy and paste

- 1) Start with any non-empty image
- 2) For each i :
 transform the image
- 3) New image = union of transformed ones
- 4) Repeat



Chaos Game

- 1) Start with any point p
- 2) Draw p
- 3) Select a random f_i
- 4) $new_p = f_i(p)$
- 5) Repeat



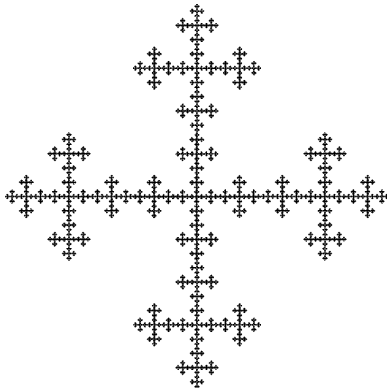
100 points

How do I generate a self-similar set?

Given: affine transformations f_i

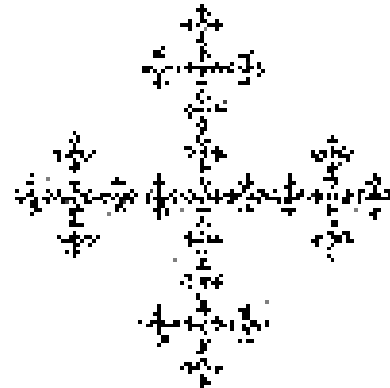
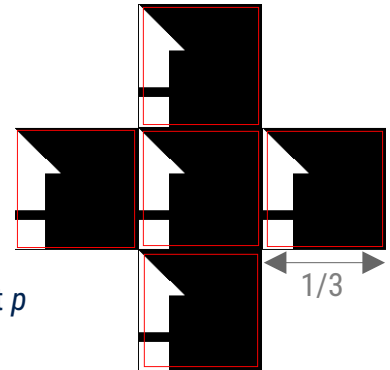
Copy and paste

- 1) Start with any non-empty image
- 2) For each i :
 transform the image
- 3) New image = union of transformed ones
- 4) Repeat



Chaos Game

- 1) Start with any point p
- 2) Draw p
- 3) Select a random f_i
- 4) $new_p = f_i(p)$
- 5) Repeat



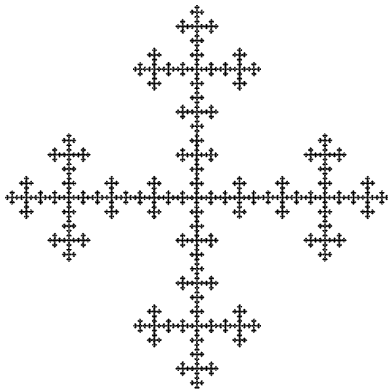
1,000 points

How do I generate a self-similar set?

Given: affine transformations f_i

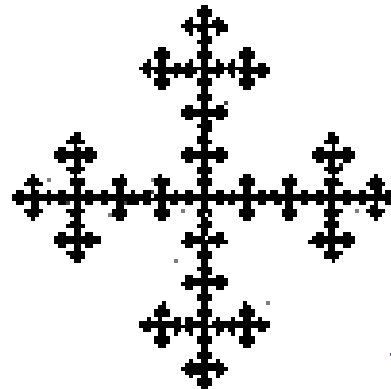
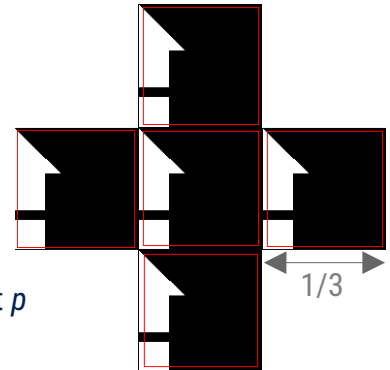
Copy and paste

- 1) Start with any non-empty image
- 2) For each i :
 transform the image
- 3) New image = union of transformed ones
- 4) Repeat



Chaos Game

- 1) Start with any point p
- 2) Draw p
- 3) Select a random f_i
- 4) $new_p = f_i(p)$
- 5) Repeat



10,000 points



See you(r fractal)
on fracmi.cc !



See you(r fractal)
on fracmi.cc !